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DIFUSSION COEFFICIENT OF A PLASMA IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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6

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I. L. Popov

In the case of a fully ionized plasma the diffusion coefficient in the direction perpendicular to magnetic field has the form /221[*]

$$D_{\perp}=\frac{e^2 N_i \ln\lambda}{6\pi(\varepsilon\varepsilon_0)^2 B^2 u}, \tag{1}$$

where e is the electronic charge, N_i is the ion density, u is the mean thermal velocity, ε is the apparent dielectric permittivity of the plasma, ε_0 is the absolute dielectric permittivity of a vacuum, B is the magnetic induction, ln λ is the Coulomb logarithm.

As shown experimentally, the expression (1) gives a greatly understated value for the diffusion coefficient.

At distances on the order of the Debye screening radius, there may appear an unbalanced electric charge, and within those distances the plasma may be considered to be a unipolarly charged medium.

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*Numbers in the margins indicate pagination in the original foreign text.

As shown by the author, between the electro-hydrodynamic screening radius (the distance within which the charge density in a unipolarly charged medium changes by a factor of e) and the Debye screening radius, we have the following relationship

$$r_e = \left(\frac{ur_d}{D}\right) r_d \tag{2}$$

where r_e is the electro-hydrodynamic screening radius, r_d is the Debye screening radius, D is the diffusion coefficient.

For the case

$$\frac{ur_d}{D} = 1 \tag{3}$$

the expressions for the Debye and electro-hydrodynamic screening radii become identical.

Let us find the diffusion coefficient in Equation (3):

$$D = ur_d = \sqrt{\frac{\varepsilon\varepsilon_0}{\varrho}\frac{kT}{e}}. \tag{4}$$

Using the Einstein relation

$$\frac{D}{b} = \frac{kT}{e}, \tag{5}$$

from Equation (4) we get an expression for the ion mobility /222

$$b = \sqrt{\frac{\varepsilon\varepsilon_0}{\varrho}}. \tag{6}$$

In extreme cases a plasma may be considered a gas, and then Equation (6) may be used to calculate the ion mobility in a liquid.

For the case when the electric field changes relatively slowly (the frequency of the field change is less than the Larmor ion frequency), the dielectric permittivity of the plasma in crossed electric and magnetic fields is given by

$$\varepsilon = 1 + \frac{Q}{\varepsilon_0 B^2}. \tag{7}$$

For a plasma whose density is not very low

$$\frac{Q}{\varepsilon_0 B^2} \gg 1. \tag{8}$$

Therefore, (4) implies

$$D_\perp = \frac{kT}{Be}. \tag{9}$$

Equation (9) gives an expression for the ion mobility

$$b = \frac{1}{B}. \tag{10}$$

By generalizing the expression for the ion mobility in gases, obtained by the author in [1], to the case of a plasma, we find

$$b = \frac{1}{B}\left[1 + \frac{E}{uB} + \left(\frac{E}{uB}\right)^2\right]. \tag{11}$$

In Equation (11), E is the intensity of the electric field in a plasma.

Equations (5), (11) give us an expression for the diffusion coefficient

$$D = \frac{kT}{Be}\left[1 + \frac{E}{uB} + \left(\frac{E}{uB}\right)^2\right]. \tag{12}$$

A magnetic field will be called "strong" if it satisfies the relation

$$B = \sqrt{\frac{Nm_e}{\varepsilon_0}}, \tag{13}$$

where $N = N_i$ is the particle density in the plasma, m_e is the electron mass.

An electric field will be called "strong" if

$$E = \sqrt{\frac{NkT}{\varepsilon_0}}. \tag{14}$$

Equations (12) - (14) imply that

$$D = \frac{kT}{Be}\left(1 + \sqrt{\frac{m_i}{2m_e}} + \frac{m_i}{2m_e}\right). \tag{15}$$

For the case

$$B = \sqrt{\frac{Nm_i}{\varepsilon_0}}, \quad E = \sqrt{\frac{Nm_e c^2}{\varepsilon_0}}, \tag{16}$$

where m_i is the ion mass, c is the speed of light, Equations /223
(13), (14), (16) yield

$$D = \frac{kT}{Be}\left[1 + \sqrt{\frac{m_e}{m_i}} \cdot \frac{c}{u} + \frac{m_e c^2}{m_i u^2}\right]. \tag{17}$$

Since

$$u = \sqrt{\frac{m_e}{m_i}}\, c, \tag{18}$$

then Equation (17) gives

$$D = \frac{3kT}{Be}. \tag{19}$$

For the case

$$\frac{kT}{Be} = \frac{\hbar}{\sqrt{m_e m_i}}, \tag{20}$$

where $\hbar$ is the reduced Planck's constant, Equation (12) gives

$$D = \frac{\hbar}{\sqrt{m_e m_i}}\left[1 + \frac{E}{uB} + \left(\frac{E}{uB}\right)^2\right]. \tag{21}$$

Inasmuch as

$$\frac{E}{uB} \ll 1,$$

Equation (21) yields

$$D = \frac{\hbar}{\sqrt{m_e m_i}}. \tag{22}$$

We use the expression in (22) to estimate the lifetime of a high temperature xenon plasma in a magnetic trap whose linear dimension is 0.5 m

$t \sim \frac{l^2}{D} = \frac{l^2\sqrt{m_e m_i}}{\hbar} = 10^6$ sec (1 year $\simeq 32 \cdot 10^6$ sec).

REFERENCES

1. Popov, I. L. Doklady Akademii Nauk URSR, Series A, 40, 1972.

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